Attribute relation learning for zero-shot classification

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In computer vision and pattern recognition communities, one often-encountered problem is that the limited labeled training data are not enough to cover all the classes, which is also called the zero-shot learning problem. For addressing that challenging problem, some visual and semantic attributes are usually used as mid-level representation to transfer knowledge from training classes to unseen test ones. Recently, several studies have investigated to exploit the relation between attributes to aid the attribute-based learning methods. However, such attribute relation is commonly predefined by means of external linguistic knowledge bases, preprocessed in advance of the learning of attribute classifiers. In this paper, we propose a unified framework that learns the attribute–attribute relation and the attribute classifiers jointly to boost the performances of attribute predictors. Specifically, we unify the attribute relation learning and the attribute classifier design into a common objective function, through which we can not only predict attributes, but also automatically discover the relation between attributes from data. Furthermore, based on the afore-learnt attribute relation and classifiers, we develop two types of learning schemes for zero-shot classification. Experimental results on a series of real benchmark data sets suggest that mining the relation between attributes do enhance the performances of attribute prediction and zero-shot classification, compared with state-of-the-art methods.

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1. Introduction

For traditional object classification problems in computer vision and pattern recognition, an intent classifier is usually designed based on a training set with samples from specific classes, and then evaluated on a test set using samples from the same categories. However, we are currently faced with more challenging problem scenarios, such as (1) multi-class classification problems where there are not enough labeled training data to cover all object classes, and (2) multi-task ones where the available training data do not provide examples of desired outputs, which are called zero-shot learning [1,2]. As a meaningful and challenging problem setting, zero-shot learning has attracted increasing attention in recent years [1–9]. It is worth noting that many design schemes for traditional classifiers cannot be directly used for zero-shot learning, due to the fact that the training and the target classes are commonly disjoint or different.

On the other hand, attributes describing objects (e.g. “red,” “stripe” and “similar to dog”) can be obtained relatively more conveniently than traditional class labels, because humans can typically provide good prior knowledge about attributes for some objects though they may be unable to name them. For example, Fig. 1 illustrates images of pandas and tigers for which we might not know their precise names (class labels) if we have never seen them before but can partly describe these objects via some of their attributes such as furry, black, big, and with ear etc. Moreover, as attributes are assigned on a per-class basis other than a per-image basis, it can significantly reduce the manual effort on adding a new object category.

Here, the term “attribute”, as defined in Webster’s dictionary, means “an inherent characteristic” of an object. For zero-shot learning problems, researchers have explored various kinds of attributes, e.g., similarity to known object categories [4,10], appearance adjectives (such as shape, texture, and color) [9,11], and the presence or absence of parts [3]. Accordingly, these attributes are classified as (1) similarity based attributes [4,10], (2) semantic attributes which can be described in language [9,11], and (3) discriminative attributes [3]. Due to the fact that attributes can be frequently shared by different objects, they can be deemed as one type of transferable knowledge [3,9]. In fact, it has also been shown that attributes behave effectively in leveraging knowledge between different object categories and compensating for the lack of labeled data [1,8,12,13], which is especially valuable for zero-shot learning problems. Consequently, a class of attribute-based methods in face verification [6], image annotation [14], image...
search and retrieval [15–18] has been developed with promising results.

Among various attribute-based approaches, the exploitation of the relationship between attribute and object class, which may convey prior information about object categories, has been proven beneficial to enhance the performance of object classification [8,19,20]. Likewise, intuitively the relationship between attributes of objects can also convey supplementary information from another aspect. As illustrated in Fig. 2, attributes corresponding to “vegetation” and “ground” are usually positively correlated, “ground” and “water” might be negatively correlated, while “swim” and “vegetation” tend to be uncorrelated. Unfortunately, in traditional attribute-based methods [3,9,14,21], the relationship between attributes is seldom considered.

As mentioned above, there exit some correlation relation (i.e., positive, negative and uncorrelated ones, etc.) between attributes. In the literature, some works have also demonstrated that mining such relation is helpful to boost the performance of attribute classifiers. For example, researchers in [4,14,19,22] have considered the attribute–attribute relation to some degree. However, the relation they adopted is largely predefined in terms of some criteria such as correlation [4,22] and mutual information [14,19], or by means of external linguistic knowledge bases. Intuitively, it will be more reasonable and more prone to be optimal if we can learn the relationship between attributes automatically from data.

In this paper, we propose to unify the attribute relation learning and the attribute classifier design into a common objective function, through which we can not only predict attributes (e.g. “white” and “not furry”), but also automatically discover the (correlation) relation between attributes (e.g. “side mirror” is highly related to “head light”). The major contribution of this paper is two-fold: (1) we propose a model called attribute relation learning (ARL) to jointly learn the attribute–attribute relation and the attribute classifiers, through which the (correlation) relation between attributes can be discovered automatically and explicitly; (2) we develop two learning schemes for attribute-based zero-shot classification based on the afore-learnt attribute relation and classifiers. To the best of our knowledge, our work is the first attempt to formulate the learning of attribute–attribute relation and attribute classifiers into a unified objective function, and to reuse such relation to boost traditional attribute classifiers that are trained separately.

The rest of this paper is organized as follows. We introduce related works on zero-shot learning, multi-task relation learning and attribute relation learning in Section 2. In Section 3, we first present our proposed model for jointly learning attribute–attribute
relation and attribute classifiers, and then introduce an alternating optimization algorithm to solve the optimization problem. Section 4 introduces the attribute-class mapping method and the proposed two types of attribute-based zero-shot classification schemes with attribute relation incorporated. Experimental results are reported in Section 5. Finally, we conclude this paper and indicate several issues for future works in Section 6.

2. Related works

In the following, we will briefly review the related works on zero-shot learning, multi-task relation learning, and then review the most relevant works on attribute relation learning.

2.1. Zero-shot learning

To the best of our knowledge, the concept of zero-shot learning can be traced back to one of the early works proposed by Larochelle et al. [1], which attempts to solve the problem of predicting novel samples that were unseen in the training categories. It is an important problem setting especially when the labeled training data are not enough to cover all the classes. Two key components are necessary in zero-shot learning [1]. One is how to generate an intermediate level representation to transfer knowledge from observed training classes to unseen target ones, and the other deals with mapping test data in the intermediate level representation to novel categories.

For the first problem, various representation of transferable knowledge have been proposed, such as distance metrics [23,24], class priors [25,26], discriminating aspects [27,28], and descriptive attributes [3,10,12,29]. Larochelle et al. [1] adopted a character font bitmap as the intermediate class description for zero-shot learning, and mapped novel classes of hand written digits into these bitmaps. However, such kind of intermediate class description cannot be generated to deal with non-trivial problems such as object recognition and scene recognition. Researchers in [2,30] proposed methods to obtain the intermediate class description such as semantic knowledge base to perform zero-shot learning. Meanwhile, in computer vision community, researchers have suggested that attributes are useful to describe familiar and unfamiliar things [3,7,12,29,31–34], to serve as mid-level features for object classification [9,10,19] and image retrieval [16,17,20,35], and to facilitate zero-shot learning paradigms [3,9,13]. Attributes have become an important way to perform zero-shot learning by transferring knowledge from training set to novel test set.

For the second problem, Lampert et al. [9] proposed an attribute-based object class model for zero-shot recognition, given the association between object categories and attributes. Informally, the attribute-based classification method models object classes through an inventory of descriptive attributes provided by human beings. They also develop two techniques to perform zero-shot learning with the help of attributes, i.e., Direct Attribute Prediction (DAP) and Indirect Attribute Prediction (IAP). It is also suggested in [9] that DAP is superior to IAP in zero-shot classification problems. Following the pioneering work of Lampert et al. [9], a number of attribute-based methods have been recently developed for addressing different kinds of applications [1,8,11,13,36].

2.2. Multi-task relation learning

Multi-task learning [37–39] learns multiple tasks jointly, seeking to improve the generalization performance of a learning task with the help of some other related tasks. A major assumption for this learning paradigm is that all tasks are related so that the joint learning is beneficial, and hence modeling the relation between tasks accurately is critical.

Recently, the so-called task relationship learning approach has been proposed to discover the relationship between tasks automatically, e.g., the multi-task Gaussian process (GP) model [40] and a regularized method [41] under the Bayesian framework. Using a task covariance matrix, this approach provides an effective way to characterize different types of pairwise task relation. To be specific, the task covariance matrix \( \mathbf{C} \) is defined as a parameter matrix for the matrix-variate normal distribution [42] over the model parameters in least square regression or support vector machine (SVM) [43]:

\[
\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_M] \sim \mathcal{N}_{D \times M}(\mathbf{W}|0_{D \times M}, \mathbf{I}_D \otimes \mathbf{C})
\]

where \( \mathbf{w}_i \in \mathbb{R}^D \) is the model parameter vector for the \( i \)th task, \( \mathcal{N}_{D \times M}(\mathbf{A}, \mathbf{C} \otimes \mathbf{I}) \) denotes a matrix-variate normal distribution with mean \( \mathbf{M} \in \mathbb{R}^{D \times M} \), row covariance matrix \( \mathbf{A} \in \mathbb{R}^{D \times D} \), and column covariance matrix \( \mathbf{C} \in \mathbb{R}^{M \times M} \). The probability density function of the matrix-variate normal distribution is as follows:

\[
p(\mathbf{W}|\mathbf{M}, \mathbf{A}, \mathbf{C}) = \frac{\exp \left( \left(1/2\right) \text{tr}(\mathbf{A}^{-1}(\mathbf{X} - \mathbf{M})(\mathbf{X} - \mathbf{M})^\top) \right)}{\sqrt{(2\pi)^D |\mathbf{A}| |\mathbf{C}|}}
\]

where \( \text{tr}(\cdot) \) denotes the trace of a square matrix, \( |\cdot| \) denotes the determinant of a square matrix, and \( \mathbf{C}^{-1} \) denotes the inverse of a non-singular matrix \( \mathbf{C} \) or the pseudo-inverse when it is singular.

Given the prior defined in Eq. (1) and the likelihood (i.e., Gaussian noise model for regression problem or logistic model for classification problem), the maximum a posteriori (MAP) solution is obtained by solving the following problem:

\[
\min_{\mathbf{W} \in \mathbb{C}} \sum_{m=1}^{M} \sum_{n=1}^{N} \left( y^n_m - f_m(\mathbf{x}^n_m) \right)^2 + \frac{\lambda}{2} \text{tr}(\mathbf{W}^\top \mathbf{W})
\]

subject to \( \mathbf{C} \succeq 0 \) and \( \text{tr}(\mathbf{C}) = 1 \) (3)

where \( \left(\cdot, \cdot\right) \) denotes the empirical loss corresponding to the likelihood, \( \lambda \) is a regularization parameter to balance the tradeoff between the empirical loss and the regularization term, and the constraint \( \mathbf{C} \succeq 0 \) in Eq. (3) is used to restrict \( \mathbf{C} \) as positive semi-definite because it denotes a task covariance matrix. The second constraint in Eq. (3) is derived from the matrix-variate normal prior in Eq. (1) and is used to regularize the task relation. One can see that the matrix \( \mathbf{C}^{-1} \) is used to model the covariance between the columns in the model parameter \( \mathbf{W} \), and hence to model the task relationship since each column in \( \mathbf{W} \) represents the model parameters of the corresponding task.

There are two advantages of the task relationship learning approach: (1) three types of relation can be modeled, i.e., positive, negative and uncorrelated relation; (2) the relationship between tasks can be learned from data automatically through the joint learning of multiple tasks. More recently, this approach has been widely applied to many domains demonstrating its effectiveness, such as multi-task learning [44–48], transfer learning [49], multi-label learning [50,51], and multi-view learning [52].

It is worth noting that the data used for tasks in multi-task learning vary from each other, while different attribute classifiers share the same data in attribute-based classification. However, to characterize the relationship between attributes, we can adopt the task relationship learning approach in multi-task learning to model the attribute-attribute relation in this work.

2.3. Attribute relation learning

As mentioned above, attributes can be used to serve as a kind of intermediate representation of transferable knowledge in zero-shot learning scenarios, which is called attribute-based classification. Among extensive study on attribute-based classification,
some researchers have demonstrated that considering the attribute-class relation which is a type of prior information can result in improved generalization performance \cite{8,14,19,20}. In contrast, less attention is focused on attribute–attribute relation learning, where attributes are expected to benefit from each other.

Recently, some works have investigated incorporating attribute–attribute relation to attribute-based classification. For example, Wang et al. \cite{19} and Kovashka et al. \cite{14} proposed models that treat object attributes and class labels as latent variables, where the relation between attributes is captured through a tree-structured graph. In this graph, vertices denote attributes and edges are restricted to pairs of attributes that have the highest mutual information. However, this type of attribute–attribute relation is pre-computed. Rohrbach et al.\cite{4} use external linguistic knowledge bases (e.g. WordNet, Wikipedia, and Yahoo Web) to mine the semantic relationship between object classes and corresponding attributes. It has been shown that web search for part-whole relationship is a better way to mine the association of attribute–attribute for object categories. Similarly, researchers of \cite{12,22} mine attribute–attribute association from language to achieve unsupervised knowledge transfer. In these works, extra expert knowledge for natural language processing is required, and the object–attribute relation other than the attribute–attribute relation learning is their focus. Siddiquie et al. \cite{20} employ structured learning to form a framework for images ranking and retrieval based on multi-attribute queries, demonstrating modeling pair-wise correlation between different attributes can lead to better performance.

Different from previous work, we use the task relationship learning approach used in multi-task learning \cite{40,41} to model the relationship between attributes. In this way, we can learn the positive, negative and uncorrelated relation between attributes automatically from data. Furthermore, we propose to reuse the discovered relation between attributes to boost the classification performance of novel object categories in zero-shot learning scenarios, by incorporating such relation into traditional classifiers, e.g., Support Vector Machine and kernel ridge regression.

3. Our attribute relation learning (ARL) approach

In this section, we will describe our proposed approach of learning the relation between attributes automatically from data in detail. As multiple attributes are usually used in the attributes-based classification, we resort to the task relationship learning approach in multi-task learning \cite{41} to model the correlation relation between attributes. We start by describing the notations used in this paper, and then propose the unified formulation to learn the attribute–attribute relation and attribute classifiers simultaneously from a multi-task learning perspective. Finally, an alternating optimization method is introduced to solve the proposed objective function.

3.1. Notations

Suppose we are given M attributes for each object class. For the m-th attribute classifier, the training set \( D_m \) consists of \( N_m \) data points \( \{x_i\}_{i=1}^{N_m} \in \mathbb{R}^{d_a} \), represented as \( D_m = \{x_1, \ldots, x_{N_m}\} \in \mathbb{R}^{d_a \times N_m} \). For each data point \( x_i \in \mathbb{R}^{d_a} \), its corresponding attribute vector label is denoted as \( y_i \in \{1, 0\} \). Note that only binary attributes are considered in this work. For each attribute classifier, we want to learn a linear function \( f_m(x) = w_m^T x + b_m \), where \( w_m \) is the weight vector and \( b_m \) is the bias term. Denote \( W = [w_1, w_2, \ldots, w_M] \) where \( w_m \in \mathbb{R}^{d_a} \) and \( b = [b_1, \ldots, b_M]^T \). Traditionally, the parameters of attribute classifiers are learned through training corresponding classifiers independently, without any attribute relation considered.

In the following, we will describe our approach where the attribute classifier and the attribute relation can be learned jointly in a unified object function regularized by a task covariance matrix.

3.2. Problem formulation

Following the notations in the previous sub-section, we propose the following objective function to learn multiple attribute classifiers jointly from a multi-task learning perspective:

\[
\min_{W} \sum_{m=1}^{M} \sum_{i=1}^{N_m} \text{Loss}(y_i, f_m(x_i^m)) + \lambda_1 \sum_{m=1}^{M} R(w_m) \tag{4}
\]

where the first term is a loss function which gives the empirical loss on the training data, and second term is a regularizer to control the complexity of weight vector \( w_m \), while \( \lambda_1 \) is a regularization parameter used to tune the tradeoff between the empirical loss and the regularization term. Actually, the model defined in Eq. (4) is a general framework for joint attribute classifiers’ learning, as one can utilize various loss functions (e.g., least squares loss and hinge loss) and various regularizers (e.g., L1-norm, L2-norm and nuclear norm) to adapt to the problems at hand. For simplicity, we adopt the least squares loss function and L2-norm regularizer in this paper. Accordingly, the problem defined in Eq. (4) can be rewritten as follows:

\[
\min_{W, b, \theta} \sum_{m=1}^{M} \sum_{i=1}^{N_m} \rho_i (y_i - (w_m^T x_i^m + b_m))^2 + \frac{\lambda_2}{2} \text{tr}(WW^T) \tag{5}
\]

s.t. \( \rho_i \geq 0 \), \( \rho_i = \rho_m \), \( \forall m, m = 1, \ldots, M \)

where \( \rho_m \) is a slack variable.

Following the work in \cite{38}, we resort to the column covariance matrix of \( W \) to model the relationship between \( w_m \)’s. Accordingly, we get the following objective function:

\[
\min_{W, b, \theta} \sum_{m=1}^{M} \sum_{i=1}^{N_m} (\rho_i y_i^m)^2 + \frac{\lambda_1}{2} \text{tr}(WW^T) + \frac{\lambda_2}{2} \text{tr}(WC^{-1}W^T) \tag{6}
\]

s.t. \( \rho_i \geq 0 \), \( \text{tr}(C) = 1 \)

where \( \lambda_1 \) and \( \lambda_2 \) are two regularization parameters and \( C \) is the column covariance matrix of the weight matrix \( W \). The constraint \( C \geq 0 \) in Eq. (6) is used to restrict \( C \) as positive semi-definite because it denotes a task covariance matrix. The second constraint in Eq. (6) is used to regularize the task relation. Here the inverse covariance matrix \( C^{-1} \) plays a role of coupling pairs of weight vectors, reflecting the attribute–attribute relationship.

To avoid the data imbalance problem partly where one attribute classifier with so many data points dominates the empirical loss, we reformulate the problem defined in Eq. (6) as follows:

\[
\min_{W, b, \theta} \sum_{m=1}^{M} \sum_{i=1}^{N_m} \frac{1}{T_m} (\rho_i y_i^m)^2 + \frac{\lambda_1}{2} \text{tr}(WW^T) + \frac{\lambda_2}{2} \text{tr}(WC^{-1}W^T) \tag{7}
\]

s.t. \( \rho_i \geq 0 \), \( \text{tr}(C) = 1 \)

In the attribute-based zero-shot learning scenarios, all attributes usually share a same training set. In such cases, the problem defined in Eq. (7) can be rewritten as follows:

\[
\min_{W, b, \theta} \sum_{m=1}^{M} \sum_{i=1}^{N} (\rho_i y_i^m)^2 + \frac{\lambda_1}{2} \text{tr}(WW^T) + \frac{\lambda_2}{2} \text{tr}(WC^{-1}W^T) \tag{8}
\]

s.t. \( \rho_i \geq 0 \), \( \text{tr}(C) = 1 \)

which is called attribute relation learning (ARL) model in this paper. Through this model, we can obtain not only the parameters
of all attribute classifiers but also the attribute–attribute relation explicitly and automatically.

It is easy to find that the problem of Eq. (8) is jointly convex with respect to $W$, $b$ and $C$. For reasons of efficiency, an alternating optimization method is adopted to optimize this model, with more details described in the following.

3.3. Alternating optimization algorithm

Now we are in the position of considering the optimization of the model defined in Eq. (8) with three variables to be optimized. As the variables in Eq. (8) are jointly convex, the problem can be solved by an alternating optimization method. The first step is to optimize $W$ and $b$ given a fixed $C$, and the second step is to optimize $C$ when $W$ and $b$ are fixed.

3.3.1. Optimizing $W$ and $b$ when $C$ is fixed

Given a fixed $C$, the problem defined in Eq. (8) can be presented in the following form:

$$
\min_{w,b} \frac{1}{N} \sum_{i=1}^{N} \sum_{m=1}^{M} \lambda_i w_i \frac{1}{2} tr(W^T W) + b_m \frac{1}{2} tr(W C^{-1} W^T) + \frac{1}{2} \sum_{i=1}^{N} \alpha_i y_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{m=1}^{M} \alpha_i b_m x_i^T W C^{-1} x_i
$$

subject to $y_i - b_m x_i^T w_i = 0$, $m = 1, ..., M$ \quad (9)

With the well-known kernel trick [53], it is easy to show that the dual form of Eq. (9) can be written as follows:

$$
\min_{\alpha} \frac{1}{2} \alpha^T K \alpha - \sum_{i=1}^{N} \sum_{m=1}^{M} \alpha_i m_i \mu_i m_i - y_i \sum_{i=1}^{N} \alpha_i x_i^T K \alpha_i + b_m \mu_i m_i \quad (10)
$$

where $\alpha = (\alpha_1, ..., \alpha_N, \mu_1, ..., \mu_M)^T$, and $\Lambda$ is diagonal with elements value $N$ if the corresponding data point belongs to the $m$-th attribute classifier. Note that $K = K + (1/2)\Lambda$, and $K$ is the kernel matrix on all data points for all attributes classifiers with element

$$
k(x_i^m, x_j^m) = e_i^m C(i, j) + \lambda_2 \mu_i \mu_j \quad (11)
$$

where $e_i^m$ is the $m$-th column vector of $M \times M$ identity matrix $I$.

Now we are in the position of considering the optimization of the model defined in Eq. (8) with three variables to be optimized. As the variables in Eq. (8) are jointly convex, the problem can be solved by an alternating optimization method. The first step is to optimize $W$ and $b$ given a fixed $C$, and the second step is to optimize $C$ when $W$ and $b$ are fixed.

4. Incorporating attribute relation into zero-shot classification

As mentioned in [9], there are two major components in zero-shot learning. The first one concerns generating an intermediate level representation, e.g., attributes which are employed in this work. And the second one is to map test data points represented in the mid-level form to novel categories. In this paper, we adopt the probabilistic Direct Attribute Prediction (DAP) model introduced in [9] to perform such attribute–class mapping. In the following, we first briefly review the scheme of DAP, and then propose two schemes for attribute-based zero-shot classification by reusing the attribute relation learned from the proposed ARL method.

4.1. DAP Model

For zero-shot learning problems where the training and test classes are disjoint, we denote the $K$ training classes as $S = \{s_i\}_{i=1}^K$ and the $L$ novel test classes as $U = \{u_i\}_{i=1}^L$. If for all the training and test classes an attribute representation $a \in \mathbb{Y}$ is available, we can learn a non-trivial classifier $a: \mathbb{X} \rightarrow \mathbb{S}$ by transferring information between $S$ and $U$ through $Y$.

As one general method to integrate attributes into multi-class classification, the scheme of DAP model is illustrated in Fig. 4. It uses a mid-level layer of attribute variables to decouple the inputs from the layer of class labels. In the training process, the output class label of each sample induces a deterministic labeling of the attribute layer, where the parameters of each attribute classifiers can be determined. In the test process, the attribute values are predicted by the trained attribute classifiers, from which the test class labels can be inferred. In the following, we explain the graphical structure in Fig. 3 in a probabilistic way [9].

Given a set of training data $D = \{x_i\}_{i=1}^N$ where $x_i \in \mathbb{R}^d$, and an attribute representation $a = (a_1, a_2, ..., a_M)$ for any training class $s$, we can learn $M$ probabilistic classifiers for each attribute $a_m$, and compute the posterior probability $p(a_m | x)$ of this attribute. Then the complete image–attribute layer can be expressed as

![Fig. 3. Direct Attribute Prediction model for zero-shot classification.](image-url)
\[ p(a|\mathbf{x}) = \prod_{m=1}^{M} p(a_m|\mathbf{x}) \] which is provided by the learnt attribute classifiers. In the test process, we assume that every test class \( u \) induces its attribute vector \( a^u \) in a deterministic way, i.e. \( p(a|u) \) is equal to 1 if \( a = a^u \) and 0 otherwise.

Under Bayes’ rule, the attribute–class layer can be expressed as
\[ p(u|x) = \frac{p(u)p(x|u)}{p(x)} = \frac{p(u)}{p(a^u)} \prod_{m=1}^{M} p(a_m^u|x) \]
(14)

If there is no extra prior information, we assume that all class priors \( p(u) \) are equal, which allows us to ignore the factor \( p(u) \) in the following. We assume the factor \( p(a^u) \) follows a factorial distribution \( p(a^u) = \prod_{m=1}^{M} p(a_m^u) \). The empirical mean
\[ p_\text{emp}(a_m^u) = \frac{1}{K} \sum_{k=1}^{K} a_m^u \]
over all training classes can be used as attribute prior where \( s_k \) is the \( k \)-th class of the training set. Finally, a test image \( z \) is classified as the best output class from all test classes \( u_1, \ldots, u_t \) using the maximum a posteriori (MAP) prediction in the following form:
\[ \text{Label}(z) = \arg \max_i \prod_{m=1}^{M} \frac{p(a_m^u|z)}{p(a_m^u)} \]
(15)

The DAP model enables us to transfer the attribute–class association of known classes to unseen ones, and facilitates the realization of zero-shot learning system.

4.2. Proposed zero-shot classification Schemes

In Section 3, we have proposed the ARL model to discover the underlying relation between attributes automatically from data. After obtaining such relation, we also want to reuse it to boost the attribute prediction performances of traditional attribute classifiers that do not consider such relation. In this sub-section, based on different ways of utilizing the attribute relation, we propose two types of schemes for attribute-based zero-shot classification under the DAP model, including: (1) ARL (i.e. ARL+DAP) scheme, and (2) ARL+SVM/KRR (i.e. ARL+SVM/KRR+DAP) scheme. The first scheme is to consider the attribute relation during the learning of attribute classifiers. And the second one is to modify the outputs of traditional attribute classifiers using the discovered attribute relation learned from the ARL model.

4.2.1. ARL scheme

The ARL scheme uses our ARL model to learn attribute classifiers, and the DAP technique to realize the attribute-label mapping. Fig. 4 illustrates the overview of this scheme. As illustrated in Fig. 4, we first use all the training images and their corresponding attributes to train \( M \) attribute classifiers jointly using the proposed ARL model. Then, the trained classifiers are employed to predict the attribute values of unseen test images, and each test image will be given \( M \) attributes of real values. It is worth noting that the trained classifiers have considered the attribute–attribute relation implicitly in the optimization process. With specific inventory of attributes for each object class, the predicted attribute values will be mapped into class labels through DAP technique.

4.2.2. ARL+SVM/KRR scheme

Given the attribute–attribute relation learnt from the proposed ARL model, we also want to reuse it to improve the performance of traditional classifiers that are learned separately, e.g. support vector machine (SVM) and kernel ridge regression (KRR). Hence, the second attribute-based zero-shot classification scheme utilizes SVM and KRR to perform attribute prediction, and uses the proposed ARL model to learn the relation between attributes. Then we modify the predicted attribute values using the learnt attribute relation explicitly, with more details described in the following. Similar to the first scheme, the DAP technique is used to perform the attribute-label mapping. Accordingly, the overview of such scheme is illustrated in Fig. 5.

There might be many ways to incorporate the attribute–attribute relation into the attribute values predicted by traditional classifiers. In this work, we utilize a simple and natural way, with the underlying intuition that attributes can benefit from each other according to their correlation relation. Specifically, the attribute value vector \( \text{Score}(z) = [\text{Score}_1 \ldots \text{Score}_M] \) for a test image predicted by SVM or KRR is modified by the attribute–attribute correlation in the following form:
\[ \text{Score}^* = (\text{Score}^T \times \text{Cor})^T \]
(16)

where \( \text{Score}^* \in \mathbb{R}^M \) and \( \text{Cor} \in \mathbb{R}^{M \times M} \) is learnt from our ARL model as described in Algorithm 1. Experiments in Section 5 demonstrate the effectiveness of the proposed method.

5. Experiments

In this section, we perform experiments to evaluate the proposed methods on three challenging data sets (Animals with Attributes [9], aPascal and aYahoo [3]) and our proposed Texture with Attributes data set. Two sets of experiments are carried out: (1) attribute prediction experiments, which evaluate the attribute classifiers of our ARL method learnt jointly incorporated with attribute relation; (2) zero-shot classification experiments, which evaluate the performances of proposed two types of zero-shot classification schemes with attribute–attribute relation considered. Section 5.1 introduces the data sets used in the experiments. The experiment setup is
described in Section 5.2. Finally, we report the experimental results in Section 5.3 and Section 5.4.

5.1. Data sets

In this sub-section, we will introduce the data sets with attribute description used in the experiments and their corresponding feature representation.

5.1.1. Animals with Attributes (AWA)

We use a subset of the Animals with Attributes (AWA) dataset [9], which consists of 30,475 images of 50 animals categories and 85 attributes. These attributes describe properties of animals such as color, patterns, size, anatomy, behavior etc. Since some attributes are not discriminate based on the given features, as is reported in [9], only 59 attributes are employed in our experiments. For computational reasons, we down-sample the training set to 100 images per category as the data set for our experiments, with an exception that “mole” has only 92 images. As given in [9], six types of pre-extracted feature representation, i.e. RGB color histograms (Cq), Sift, rgbShift, Phog, Surf and local self-similarity histograms (LSS) are used as descriptors for these images. And 10 classes are selected as the test set and the other 40 classes as the training set with the same division in [9]. Both attribute prediction and zero-shot classification experiments are performed on this data set.

5.1.2. aPascal and aYahoo

We also use the a Pascal-train and a Yahoo-test data sets introduced by Farhadi et al. [3]. The former consists of 20 classes and the latter 12, including animals, vehicles, and household items etc. There are 64 attributes for these two data sets, including shape, textures, anatomy and parts. We use color features as image descriptor as is given in [3]. Similarly, we use 100 images for each class to form the data set used in our experiments. Two different groups of these two data sets are used. One employs all samples in a Pascal as training set and those in a Yahoo as test set, with “aPascal & a Yahoo” for short. The other uses 5 classes (bus, car, cat, dog and motorbike) as test data while the other 15 ones in a Pascal as training data, with “aPascal” for short. Note that instances of a specific class in these two data sets may have different inventories of attributes, i.e., a unique inventory of attributes for each class cannot be obtained. Therefore, we just perform attribute prediction experiments other than zero-shot classification experiments on these two data sets, which are different from that of the AWA data set.

5.1.3. Texture with Attributes (TWA)

To facilitate study of a wide range of texture analysis problems, researchers have collected a large number of textures, both in the form of surface textures and natural scenes. Currently, there are several well-known texture data sets, such as Brodatz [55], MeasTex [56], VisTex [57] and Outex [58]. Among them it is revealed that surface textures in Outex exhibit well defined
variations to a given reference in terms of illumination, rotation and spatial resolution [58]. However, these data were not useable in attribute-based texture classification context so far, because no attributes are defined for each texture category at all. To overcome this problem, we construct the Texture with Attributes (TWA) data set and perform zero-shot classification experiments on it.

Motivated by the hierarchical structure of the ImageNet Large Scale Visual Recognition Competition 2010 (ILSRC10) [59], we design a set of seven semantic or descriptive attributes: “man-made”, “textile”, “vegetable”, “to wear”, “rectangle pattern”, “smooth” and “furry”. The attribute tree is constructed to ensure that each texture class is described by a unique attribute description set, as illustrated in Fig. 6.

Fig. 7. Attribute correlation coefficient matrix earned by different methods on aPascal (a) the proposed ARL and (b) NormMI. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)
These attributes generally correspond to basic surface texture commonly found in daily life. The values of these attributes are binary. If a texture class is positive or negative in some attribute, its value is 1 or 0 respectively. Example images of eight texture classes (Knit, Carpet, Brick, Pebble, Wood, Bark, Fur and Water) are collected from surface textures in the Outex data set, which range from man-made textures to artificial ones. The remaining collection is composed of 320 images with 40 ones for each class. Zero-shot classification experiments are performed on this data set.

For the AWA, aPascal and aYahoo data sets, features of objects are represented by histograms of local image, which are usually high dimensional. In this paper, we compute kernel matrices which are based on $\chi^2$-kernel [60] of individual feature type (sum of individual $\chi^2$-kernel when use all feature types), as done in [9]. For the bandwidth parameters of $\chi^2$-kernels, we first compute the median of the $\chi^2$-distances over the training samples, and then set bandwidth as the five times inverse of such median. Among various texture feature extraction methods, filter bank methods are most commonly used, such as wavelet transforms [61], Gabor filters [62] and contoured transforms [63] with the aim to enhance edges and lines of different orientations and scales. For the TWA data set, a 4-level decomposition structure for the Wavelet Package Transform [61] is utilized, which is a classical method for texture feature extraction. The mean and the variance of each coefficients matrix in the 4-th level form the 34-dimensional feature vector for each texture images.

### 5.2. Experimental setup

For the first set of experiments, i.e. attribute prediction, we compare the proposed ARL model with SVM as in [4,9] and KRR that are trained separately without considering any attribute–attribute relation. As for the second one, i.e. zero-shot classification experiments, we compare the proposed ARL scheme, “ARL+ SVM” and “ARL+ KRR” methods with SVM and KRR schemes, where SVM and KRR schemes use traditional SVM and KRR to train attribute classifiers respectively while DAP is used to perform the attribute–class mapping.

For attribute–attribute relation learning, we also compare the proposed ARL method to normalized mutual information (NormMI), which is a commonly used method [14,19]. Accordingly, we incorporate the attribute relation mined by NormMI into SVM and KRR, and get two zero-shot classification scheme called “NormMI + SVM” and “NormMI + KRR” respectively. In the zero-shot classification experiments, “NormMI + SVM” and “NormMI + KRR” are compared with our proposed ARL, “ARL + SVM” and “ARL + KRR” methods.

For the proposed ARL method, we can select the optimal regularization parameters $\lambda_1$ and $\lambda_2$ through grid search in the range $(0, 1]$. For computational reasons, we empirically set the regularization parameters $\lambda_1$ and $\lambda_2$ of the proposed ARL method as 0.1 and 0.05, respectively. Similar to [9], the parameter C of SVM is set to 10. The parameter $\chi$ of KRR is selected from $\{0.001, 0.01, 0.1, 1, 10, 100\}$ on the validation set of training classes. As in [9], a sigmoid transform [64] maps the outputs of attribute classifiers into probabilistic scores for DAP, where optimal values for parameter $A$ and $B$ are set on the same validation set. We adopt the classification accuracy to evaluate the attribute prediction and the zero-shot object classification performances of different methods.

### 5.3. Results of attribute prediction experiments

In this sub-section, we first show the attribute correlation learned from our proposed ARL method compared with that of normalized mutual information (NormMI), and then report the performances of the learned attribute classifiers with comparison to those of SVM and KRR.

#### 5.3.1. Learnt attribute relation

Firstly, in Fig. 7, we depict the correlation coefficient matrix among attributes learnt from the proposed ARL method and NormMI on the aPascal data set. Note that the attributes of this data set are grouped according to parts of object classes (e.g. people, vehicles and horses). Here, red and yellow indicate high correlation coefficients while blue and green denote low ones.

From Fig. 7(a), one can find that some attributes, such as “Wheel”, “Door”, “Headlight”, “Taillight” and “Side mirror”, are highly positively correlated. These attributes are all relevant to the object “vehicles”, demonstrating the attribute relation we learn is reliable. However, for the relation learnt by NormMI shown in Fig. 7(b), this trend is not so distinct. In addition, on this data set, the negative correlation between attributes can be reflected by the proposed ARL method, but cannot be reflected by NormMI. In addition, in Section 5.4, we will further investigate whether all kinds of correlation can help improve the performance of attribute-based zero-shot classification.

In addition, we analyze the convergence of the proposed Algorithm 1. The change of the objective function value defined in Eq. (8) on the aPascal data set is given in Fig. 8. From Fig. 8, one can find that objective function value decreases rapidly that takes within 100 iterations. It illustrates the fast convergence of the proposed Algorithm 1. In the following experiments, we set the iteration number as 100 empirically.

#### 5.3.2. Results of attribute prediction

In attribute-based zero-shot classification problems, higher accuracy of attribute prediction in the image–attribute layer will promote the performance of zero-shot classification in the attribute–class layer. Now, we investigate the performance of the proposed ARL method in attribute prediction in comparison with traditional SVM and KRR that consider no attribute relation. Note that individual attribute classifiers are trained on the training set, and are evaluated on the novel test set. The overall average attributes prediction accuracies among all attributes on different

![Fig. 8. Convergence of objective function value on aPascal data.](image-url)

<table>
<thead>
<tr>
<th>Methods</th>
<th>aPascal</th>
<th>aPascal &amp; aYahoo</th>
<th>AWA</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVM</td>
<td>89.67</td>
<td>88.04</td>
<td>75.26</td>
</tr>
<tr>
<td>KRR</td>
<td>86.75</td>
<td>85.62</td>
<td>73.04</td>
</tr>
<tr>
<td>ARL (ours)</td>
<td>90.53</td>
<td>89.86</td>
<td>78.79</td>
</tr>
</tbody>
</table>
data sets are reported in Table 1, where bold results illustrate the winning method.

From Table 1, one can see that the proposed ARL model consistently achieves better performance comparing to SVM and KRR on three data sets. For example, on the aPascal data set, the average attribute prediction accuracy obtained by the proposed ARL method is 90.53%, which is much higher than those of SVM (89.67%) and KRR (86.75%). This validates our intuition that learning the attribute–attribute relation automatically from data can promote the attribute prediction performance.

In addition, we report the prediction accuracies of all attributes on AWA data set in Fig. 9. From Fig. 9, one can see that, on most attributes, the proposed ARL method with attribute relation incorporated usually outperforms SVM and KRR that do not considering the relation between attributes. In addition, Fig. 9 shows that attributes such as “white” and “group” cannot generalize well from training classes to novel test ones, while others such as “hand” and “ground” can generalize well. It suggests that some attribute characterization is less informative for novel test classes.

5.4. Results of zero-shot classification

In the following experiments, we first compare the proposed ARL scheme with SVM and KRR in attribute-based zero-shot classification. Then, we incorporate the attribute–attribute relation learnt from the proposed ARL model into SVM and KRR, in comparison with the relation learnt by normalized mutual information (NormMI) as in [19]. The zero-shot classification experiments are performed on the AWA and the TWA data sets.

5.4.1. Results on AWA data set

Now we first perform zero-shot classification on the AWA data set by comparing SVM, KRR and the proposed ARL schemes. Table 2 reports the average classification accuracy of 10 test classes using different types of features.

From the results shown in Table 2, one can see that the proposed ARL method outperform SVM and KRR in predicting novel object classes, with the highest accuracy (37.40%) using all six features. These results further validates that mining the relation between attributes from data indeed boosts the performances of zero-shot object classification tasks.

Furthermore, to boost traditional attribute classifiers that are trained separately without considering the relationship between attributes, we reuse the attribute relation learned from the proposed ARL method to modify the predicted attribute values. To be specific, as mentioned in Section 4.2, the proposed “ARL+” SVM and “ARL+” KRR schemes, which are incorporated by the attribute relation discovered by the proposed ARL model, are compared with “NormMI+” SVM and “NormMI+” KRR where the attribute relation is discovered by normalized mutual information. Fig. 10 shows the zero-shot classification accuracy of SVM and KRR as well as their corresponding counterparts with different attribute relation incorporated on AWA data set. Note the term “−all” means all correlation coefficients (positive, negative and uncorrelated ones) are incorporated into the outputs of attribute classifiers, while the term “−pos” means the negative correlations are ignored.

From Fig. 10(a), one can see that, in most cases, incorporating the positive attribute correlation other than all correlation into traditional SVM usually boost the zero-shot classification results,

![Fig. 9. Attribute prediction accuracy obtained by SVM, KRR and ARL on AWA.](image)
whatever the attribute relation is learnt by NormMI and the proposed ARL method. To be specific, the “ARL-pos + SVM” scheme usually outperforms SVM and “ARL-all + SVM”, while “NormMI-all + SVM” usually outperforms SVM and “NormMI-all + SVM”. On the other hand, one can find that the “ARL-pos + SVM” achieves better performance than “NormMI-pos + SVM” in most cases, illustrating that the attribute relation learnt from the proposed ARL model is more likely to reflect the true structure among attributes. The similar trend can be found in Fig. 10(b) as in Fig. 10(a), i.e. the proposed “ARL-pos + KRR” is superior to the other four methods in most cases. These results validate the effectiveness of the proposed method we incorporate the attribute relation to zero-shot learning in Eq. (15).

### 5.4.2. Results on TWA data set

Moreover, we perform zero-shot classification experiments on TWA data set. In Section 5.4.1, it has been shown that positive attribute relation can largely promote the performances of zero-shot classification comparing to all relations. Hence, we incorporate the positive attribute relation learned by the proposed ARL as well as the NormMI to the “ARL+ SVM” and “ARL+ KRR” schemes on this data set. Due to the relatively smaller class number of this data set, a leave-two-class-out strategy is employed to perform zero-shot classification experiments. To be specific, each time samples of two classes are selected as the test data while the others are used as the training data, ensuring that the training and the test data are disjoint. Respect to eight texture classes, the number of combination is 28. The average accuracy is computed as the final results, reported in Table 3.

From Table 3, it can be seen that the proposed ARL method considering the attribute relation is superior to SVM and KRR on the TWA data set. In addition, the proposed “ARL-pos + SVM” method outperforms “NormMI-pos + SVM”, and the proposed “ARL-pos + KRR” outperforms “NormMI + KRR-pos”, respectively. It demonstrates that the attribute relation learned by the proposed ARL model is more suitable to boost the performances zero-shot classification, compared to that learned by NormMI.

### 6. Conclusion and future works

In this paper, we mainly focus on learning the relationship between attributes automatically from data for attribute-based zero-shot object classification. We first formulate the attribute–attribute relation learning and the attribute classifiers’ training in a unified objective function, which can be cast as a multi-task learning problem. In this way, the attribute–attribute relation can be discovered automatically and explicitly, with no need for external linguistic knowledge base and human intervention. Then, we propose two types of attribute-based zero-shot object classification schemes considering the attribute–attribute relation, where the learnt relation is reused to improve traditional classifiers that are trained separately without considering the underlying the relation between attributes. Finally, we evaluate the proposed methods through attribute prediction experiments and zero-shot classification experiments on three challenging data sets.

In the proposed ARL model, we using the least square loss function and L2-norm regularizer in the objective function. It may be promising to use the other forms of loss function and regularization, which will be one of our future works. Meanwhile, the semantic attribute space can be expanded by generating new attributes based on the attribute relation we learned, which may further boost the zero-shot learning performance. Also it is interesting to investigate whether using preferable attributes for specific categories can improve the performance of zero-shot learning. In addition, feature selection for attribute classifiers may be necessary and meaningful when feature dimension is high, and will also be our future work.

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**Table 3**

Zero-shot learning accuracy on TWA (%)..

<table>
<thead>
<tr>
<th>Methods</th>
<th>Average accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVM</td>
<td>62.45</td>
</tr>
<tr>
<td>NormMI-pos + SVM</td>
<td>60.32</td>
</tr>
<tr>
<td>ARL-pos + SVM (ours)</td>
<td>66.65</td>
</tr>
<tr>
<td>KRR</td>
<td>60.21</td>
</tr>
<tr>
<td>NormMI-pos + KRR</td>
<td>63.17</td>
</tr>
<tr>
<td>ARL-pos + KRR (ours)</td>
<td>65.05</td>
</tr>
<tr>
<td>ARL (ours)</td>
<td>65.89</td>
</tr>
</tbody>
</table>

**Fig. 10.** Zero-shot learning accuracy of SVM and KRR with attribute relation incorporated on AWA (a) SVM and (b) KRR.
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References


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